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Integrity Rules for Multiargument Relationships in Possibilistic Databases

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Abstract. *The paper contains an analysis of multiargument relationships in possibilistic databases. A multiargument relationship may be formally presented using the relational notation* $R(X_1, X_2,...,X_n)$*, where R is the name of the relationship and attributes* X_i *denote keys of entity sets which participate in it. The dependencies between all n attributes describe the integrity constraints and must not be infringed. They constitute a restriction for relationships of fewer attributes. In the paper it is considered a possible coexistence of associations between* $k < n$ *attributes of the n-ary relationship. The analysis is carried out using the theory of fuzzy functional dependencies. The notion of functional dependency has been appropriately extended according to the representation of data.*
Keywords: *Fuzzy databases, fuzzy sets, fuzzy functional dependencies, possibility distributions, inference rules.*

1. Introduction

Fuzzy database models have been created for managing and retrieving imperfect information. Numerous works discuss how uncertainty existing in databases should be handled. Some authors proposed incorporating fuzzy logic into data modeling techniques. So far, a great deal of effort has been devoted to the development of fuzzy data models [1, 2, 3, 4, 5, 6, 7]. There are two major approaches concerning fuzzy data representation, namely, the similarity-based approach [8] and the possibility-based approach [9]. In the former domains of attributes are associated with similarity relations and attribute values can be ordinary subsets of their domains. In the latter approach, which is applied in the presented paper, attribute values are expressed by means of possibility distributions.

The theory of possibility was introduced by Zadeh [10]. The concept of a possibility distribution is defined with the use of the concept of a fuzzy set. A fuzzy set is a generalization of an ordinary set. In classical set theory one can define a characteristic function which indicates membership of elements in sets. It is a mapping $\Re \rightarrow \{0, 1\}$, where $\Re$ denotes the universe of discourse. The characteristic function of the set A takes the value 1 if the element e belongs to A and 0 in the opposite case. However, if there are no sharp boundaries of membership the unique qualification of elements is not always obvious. Thus the set $\{0, 1\}$ should be replaced with the interval [0,1]. The definition contains a membership function which is a mapping $\Re \rightarrow [0, 1]$.

Definition 1 *Let $\Re$ be a universe of discourse. A fuzzy set A in $\Re$ is defined as a set of ordered pairs:*

$$A = \{< x, \mu_A(x) >: x \in \Re, \quad \mu_A(x) : \Re \rightarrow [0, 1]\} \ , \tag{1}$$

where $\mu_A(x)$ is a membership function.

Based on the concept of the fuzzy set one can define the concept of the possibility distribution.

Definition 2 *Let $\Re$ be a universe of discourse, X be a variable on $\Re$ and F be a fuzzy set with the membership function $\mu_F(x)$. The possibility distribution of X with respect to F is defined as*

$$\Pi_X = \{\pi_X(x)/x : x \in \Re, \quad \pi_X(x) = \mu_F(x)\}. \tag{2}$$

In the possibilistic database framework each value x of an attribute X is assigned with a number $\pi_X(x)$ from the unit interval which expresses the possibility degree of its occurrence. The possibility distribution takes the form: $\{\pi_X(x_1)/x_1, \pi_X(x_2)/x_2, ..., \pi_X(x_n)/x_n\}$, where x_i is an element of the domain of X. At least one value must be completely possible i.e. its possibility degree equals 1. This requirement is referred to as the normalization condition. Different ways of determination of the possibility degree have been described in [6]. Possibilistic databases allow for a unified way of representing of precise and ill-known information. A precise attribute value is represented by $\{1/x\}$. A fuzzy value expressed by a fuzzy set A with the membership function $\mu_A(x)$ is represented by the possibility distribution such that $\pi_X(x) = \mu_A(x)$.

The main constructs of the entity-relationship data model are entities, relationships and their attributes. An entity is an existing object which is distinguishable from other objects. A relationship is an association between entity sets. In database models, usually binary relationships occur between entity sets. However, when designing it may be necessary to define relationships comprising more sets. Relationships which involve three or more entity sets are referred to as n-ary or multiargument relationships. The number n is called the degree of the relationship. Within such connections there may exist relationships comprising fewer than n sets. However, there is no complete arbitrariness [11, 12, 13]. The relationships "embedded" in n-ary relationships are subjected to certain restrictions. They cannot be in conflict with requirements of the n-ary relationship. Cardinalities of binary relationships embedded in ternary relationships were considered in [11]. For various types of ternary relationships the authors formulated the rules to which the binary relationships between pairs of sets are subjected.

The aim of the paper is to analyze multiargument relationships in possibilistic databases. The analysis is carried out with the use of functional dependencies (FDs) which reflect integrity constraints and should be studied during the design process [14]. The classical definition of functional dependency between attributes X and Y is based on the assumption that the equality of attribute values may be evaluated formally with the use of two-valued logic. The existence of FD $X \to Y$ means that X-values uniquely determine Y-values. If attribute values are imprecise, one can say about a certain degree of the dependency $X \to Y$. The classical notion of functional dependency has to be modified according to the representation of fuzzy data. Since the notion of FD plays an important role in the design process [15], its fuzzy extension has attracted a lot of attention. Hence, different approaches concerning fuzzy functional dependencies (FFDs) have been described in professional literature. A number of different definitions emerged [16, 17, 18, 19, 20, 21]. In further considerations we will apply the definition proposed in [16].

The paper is organized as follows. The next section presents the notion of the fuzzy functional dependency which is used in the analysis. Section 3 discusses binary relationships embedded in the ternary relationship. Generalized integrity conditions for n-ary relationships are described in Section 4.

2. Functional Dependencies in Possibilistic Databases

In conventional databases a relation r of the scheme $R(U)$ is a subset of Cartesian product of attribute domains: $r \subseteq D_1 \times D_2 \times ... \times D_n$, where U denotes a set of attributes, $U = \{X_1, X_2,...,X_n\}$ and D_i denotes a domain of X_i. Let us assume that attribute values are given by means of normal possibility distributions:

$$t(X_i) = \{\pi_{t(X_i)}(x)/x : x \in D_i\}\ , \quad \sup_{x \in D_i} \pi_{t(X_i)}(x) = 1\ ,$$

where t is a tuple of r and $\pi_{t(X_i)}(x)$ is a possibility degree of $t(X_i) = x$. Allowing for possibility distributions to appear as attribute values one arrives at fuzzy relations. A tuple of a fuzzy relation is of the form

$$t = < \Pi_{X_1}, \Pi_{X_2},\ ...\ ,\ \Pi_{X_n} >, \tag{3}$$

where Π_{X_i} is a possibility distribution of X_i on $D_i = DOM(X_i)$. Thus, a relation is a subset of Cartesian product $\Gamma_{D_1} \times \Gamma_{D_2} \times ... \times \Gamma_{D_n}$, where Γ_{D_i} is a set of possibility distributions of X_i.

Values X and Y described by means of possibility distributions can be compared with the use of the possibility measure, denoted by Pos. This measure expresses the extent to which the considered values satisfy a comparison relation. According to [9], the possibility that $\Pi_X = \Pi_Y$ equals

$$Pos(\Pi_X = \Pi_Y) = \sup_x \min(\pi_X(x), \pi_Y(x)). \tag{4}$$

The possibility measure can be applied for evaluation of closeness measure for attribute values in possibilistic databases. Let t_1 and t_2 be tuples of the relation r of the scheme $R(U)$. The closeness measure of $t_1(X_i)$ and $t_2(X_i)$, denoted by $=_c$ $(t_1(X_i), t_2(X_i))$, is as follows:

$$=_c (t_1(X_i), t_2(X_i)) = Pos(\Pi_{t_1(X_i)} = \Pi_{t_2(X_i)}) = \sup_x \min(\pi_{t_1(X_i)}(x), \pi_{t_2(X_i)}(x)) \tag{5}$$

Obviously, the degree of closeness for identical values equals 1. Finally we obtain:

$$=_c (t_1(X_i), t_2(X_i)) = \begin{cases} 1 & \text{if } t_1(X_i) \text{ and } t_2(X_i) \text{ are identical} \\ Pos(\Pi_{t_1(X_i)} = \Pi_{t_2(X_i)}) & \text{otherwise} \end{cases}$$

For complex attributes $X \subseteq U$ one must consider all the components X_i of X and apply the operation *min*:

$$=_c (t_1(X), t_2(X)) = \min_i =_c ((t_1(X_i), t_2(X_i)) \tag{6}$$

where $X_i \in X$.

In [17] the following definition of the fuzzy functional dependency (FFD) for the possibilistic fuzzy data model has been proposed:

$$\begin{aligned} &X \rightarrow_\theta Y\ (\theta \in (0,1])\ \text{ is valid in } r \quad \Leftrightarrow \\ &\text{if } \forall t_1, t_2 \in r\ \ t_1(X) = t_2(X) \Rightarrow t_1(Y) = t_2(Y) \text{ else} \\ &\min_{t_1,t_2} I(t_1(X) =_c t_2(X), t_1(Y) =_c t_2(Y)) \geq \theta, \end{aligned} \tag{7}$$

where θ denotes a degree of $X \rightarrow Y$, I is Gödel implication:

$$I_G(a,b) = 1 \quad \text{if } a \leq b \quad \text{and} \quad I_G(a,b) = b \quad \text{if } a > b. \tag{8}$$

and $=_c$ is a closeness measure based on the equality of two possibility distributions.

The author proved the soundness and completeness of the axiomatic system composed of the inference rules (extended Armstrong axioms). In further works Chen developed a testing algorithm for dependency-preserving decomposition and designed a number of fuzzy normal forms.

The degree of fuzzy functional dependency contains the information to what extent X functionally determines Y. Its existence means that close Y-values correspond to close X-values to a certain degree. If values of X are almost different the degree of $X \rightarrow Y$ should be high regardless of the closeness of Y-values. This property is not satisfied by the definition proposed by Chen. Suppose $=_c(t_1(X), t_2(X)) = 0{,}1$ and $=_c(t_1(Y), t_2(Y)) = 0$. Thus the grade of $X \rightarrow Y$ equals 0 which means a lack of dependency. In [16] Cubero and Villa proposed a definition in which the grade of FFD is determined by more than one number from the unit interval. These numbers correspond to closeness measures for both X and Y. Let $X = \{ X_i \}_{i \in I}$ and $Y = \{ X_i \}_{j \in J}$, where I, $J \in \{1, 2, ..., n\}$. Let $\alpha_X = (\alpha_{X_i})$ and $\alpha_Y = (\alpha_{Y_j})$ denote vectors of thresholds imposed on the attributes of X and Y respectively:

$$\alpha_{X_i} \leq\ =_c (t_1(X_i), t_2(X_i)) \quad \text{and} \quad \alpha_{Y_j} \leq\ =_c (t_1(X_j), t_2(X_j)) \tag{9}$$

Definition 3 *Let $R(U)$ be a relation scheme where $U = \{X_1, X_2, \ldots, X_n\}$. Let X and Y be subsets of U: $X = \{ X_i \}_{i \in I}$ and $Y = \{ X_i \}_{j \in J}$, where I, $J \in \{1, 2, ..., n\}$. Let α_X and α_Y denote vectors of thresholds (9). Y is functionally dependent on X in $\theta = (\alpha_X, \alpha_Y)$ degree, denoted by $X \rightarrow_{(\alpha_X, \alpha_Y)} Y$, if and only if for every relation r of R the following condition is met:*

$$\begin{aligned} &\text{if} \quad \forall t_1, t_2 \quad \forall i \in I\ =_c (t_1(X_i), t_2(X_i)) \geq \alpha_{X_i} \quad \text{then} \\ &\qquad \forall j \in J\ =_c (t_1(Y_j), t_2(Y_j)) \geq \beta_{Y_j}\ . \end{aligned} \tag{10}$$

Thus, closeness of X-values to the degree α_X implies closeness of Y -values to the degree β_Y. According to the definition 3 one can formulate the following set of axioms (extended Armstrong's axioms):

A1: $Y \subseteq X \Rightarrow X \rightarrow_{\alpha_X,\, \alpha_Y} Y$

A2: $X \rightarrow_{\alpha_X,\, \alpha_Y} Y \Rightarrow XZ \rightarrow_{\alpha_{XZ},\, \alpha_{YZ}} YZ$

A3: $X \rightarrow_{\alpha_X,\, \alpha_Y} Y \wedge Y \rightarrow_{\alpha_Y,\, \alpha_Z} Z \Rightarrow X \rightarrow_{\alpha_X,\, \alpha_Z} Y,$

From A1, A2 and A3 the following inference rules can be derived:

D1: $X \rightarrow_{\alpha_X,\, \alpha_Y} Y \wedge X \rightarrow_{\alpha_X,\, \alpha_Z} Z \Rightarrow X \rightarrow_{\alpha_X,\, \alpha_{YZ}} YZ$

D2: $X \rightarrow_{\alpha_X,\, \alpha_Y} Y \wedge WY \rightarrow_{\alpha_{WY},\, \alpha_Z} Z \Rightarrow XW \rightarrow_{\alpha_{XW},\, \alpha_Z} Z$

D3: $X \rightarrow_{\alpha_X,\, \alpha_Y} Y \wedge Z \subseteq Y \Rightarrow X \rightarrow_{\alpha_X,\, \alpha_Z} Z;$

D4: $X \rightarrow_{\alpha_X,\, \alpha_Y} Y \Rightarrow X \rightarrow_{\beta_X,\, \beta_Y} Y$ for $\beta_X \geq \alpha_X$ and $\beta_Y \leq \alpha_Y$.

3. Consistency rules for ternary relationships

Let us consider a ternary relationship between entity sets E_1, E_2 and E_3 in a conventional database. Let X, Y and Z denote keys of E_1, E_2 and E_3, respectively. Such relationship may be formally presented with the use of the relational notation: $R(X, Y, Z)$. Cardinality of R can be expressed as M_1:M_2:M_3, where M_1 (M_2 or M_3) denotes the number of entities of E_1 (E_2 or E_3) that can occur for each pair of entities from E_2 and E_3 (E_1 and E_3 or E_1 and E_2). For ternary relationships analysis of four possible cases is necessary: 1:1:1, M_1:1:1, M_1:M_2:1 and M_1:M_2:M_3. According to [11] cardinalities of imposed binary relationships cannot be lower than the cardinality of the ternary relationship. Therefore, for the first case, the binary relationships of any cardinalities can be imposed. In the fourth case the binary relationship may be only of the many-to-many type.

The imposition of binary relationships may be also analysed using the theory of functional dependencies. The FDs describing a relationship $R(X, Y, Z)$ are as follows:

$$XY \rightarrow Z, \quad XZ \rightarrow Y, \quad YZ \rightarrow X \tag{11}$$

These dependencies describe the integrity constraints and must not be infringed. They constitute a restriction for binary relationships. The imposition of the binary relationship is admissible if no new dependency (11) will be created. For a ternary relationship of cardinality 1:1:1 there exist three dependencies (11). Let us imposed a binary relationship represented by FD $X \to Z$. The imposition of $X \to Z$ implies $XY \to Z$ which means that integrity constraints have not been disturbed. In case of cardinality M_1:M_2:M_3, where $M_i > 1$, the dependencies (11) do not occur at all. Therefore imposition of any binary relationship is inadmissible.

Let us consider a ternary relationship $R(X,Y,Z)$ with the following FFDs:

$$XY \to_{\alpha_{XY},\, \alpha_Z} Z\,, \quad XZ \to_{\beta_{XZ},\, \beta_Y} Y\,, \quad YZ \to_{\gamma_{YZ},\, \gamma_X} X\,, \tag{12}$$

where $\alpha_{XY} = (\alpha_X, \alpha_Y)$, $\beta_{XZ} = (\beta_X, \beta_Z)$, $\gamma_{YZ} = (\gamma_Z, \gamma_Y)$.
Let us define the following fuzzy sets of attributes:

$$\mathcal{L} = \{\theta_X/X,\ \theta_Y/Y,\ \theta_Z/Z\}\,, \quad \mathcal{B} = \{\gamma_X/X,\ \beta_Y/Y,\ \alpha_Z/Z\}\,, \tag{13}$$

where

$$\theta_X = (\alpha_X + \beta_X)/2, \quad \theta_Y = (\alpha_Y + \gamma_Y)/2, \quad \theta_Z = (\beta_Z + \gamma_Z)/2\,. \tag{14}$$

Membership grades of attributes depend on levels of corresponding FFDs. If an attribute does not occur on the right side of any FFD (12) its degree of membership to $\mathcal{B}$ equals 0.

Theorem 1. *Within the fuzzy ternary relationship $R(X,Y,Z)$ with FFDs* (12) *there may exist binary relationships determined by FFDs of the form $V \to_{\phi_V,\, \phi_W} W$, where $V, W \in \{X, Y, Z\}$, if $\phi_X \geq \beta_X$ and $\phi_Y \leq \beta_Y$ for $V = X$ and $W = Y$, $\phi_X \geq \alpha_X$ and $\phi_Z \leq \alpha_Z$ for $V = X$ and $W = Z$, $\phi_Y \geq \gamma_Y$ and $\phi_X \leq \gamma_X$ for $V = Y$ and $W = X$, $\phi_Y \geq \alpha_Y$ and $\phi_Z \leq \alpha_Z$ for $V = Y$ and $W = Z$, $\phi_Z \geq \gamma_Z$ and $\phi_X \leq \gamma_X$ for $V = Z$ and $W = X$, $\phi_Z \geq \beta_Z$ and $\phi_Y \leq \beta_Y$ for $V = Z$ and $W = Y$.*

Proof. Let us consider the imposition of $X \to_{\phi_X,\, \phi_Z} Z$. Basing on A2 and D3 we obtain $XY \to_{\phi_{XY},\, \phi_Z} Z$, where $\phi_{XY} = (\phi_X, \phi_Y)$. If $\phi_X < \alpha_X$ or $\phi_Z > \alpha_Z$, the integrity constraints (12) will be disturbed. The membership degrees of Z in the fuzzy sets (13) will change. In the opposite case i.e. if $\phi_X \geq \alpha_X$ and $\phi_Z \leq \alpha_Z$ the imposition is allowed because then $XY \to_{\alpha_{XY},\, \alpha_Z} Z \Rightarrow XY \to_{\phi_{XY},\, \phi_Z} Z$ (rule D4). Thus, the level of the imposed FFD is limited by values of α, β and γ in (12). The proof for other impositions is similar. □

4. General Integrity Constraints for *n*-ary Relationships

The obtained in the previous section result can be generalized for n-ary relationships. Let us denote by U the set of all attributes, $U = \{X_1, X_2, \ldots, X_n\}$, of the relation scheme $R(X_1, X_2, \ldots, X_n)$. The FDs describing n-ary relations may be presented as follows:

$$U - \{X_i\} \rightarrow X_i, \quad i = 1, 2, \ldots, n \quad . \tag{15}$$

Cardinality of an n-ary relationship between n entity sets $E_1, E_2, \ldots, E_n$ may be presented as $M_1 : M_2 : \ldots : M_n$, where M_i denotes the number of entities e_i of E_i that can occur for each (n-1)-tuple $(e_1, e_2, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n)$. Functional dependencies (15) constitute a restriction for (n-1)-ary relationships. Their quantity depends on the cardinality and is equal to the number of M_i values such that $M_i = 1$. For a relation of cardinality 1:1: ... :1 this number is n. If $M_i > 1$ for every i, the dependencies (15) do not occur at all. The FDs describing relations between (n-1) attributes may be presented as follows:

$$U - \{X_i, X_j\} \rightarrow X_i, \quad i \neq j, \quad i, j = 1, 2, \ldots, n \quad . \tag{16}$$

The imposition of an FD (16) is inadmissible if it results in creation of a new FD (15). Basing on Armstrong's axioms we obtain $U - \{X_i, X_j\} \rightarrow X_i \Rightarrow U - \{X_i\} \rightarrow X_i$. If X_i occurs in the right side of any FD (15) the considered imposition is admissible.

Corollary 1. *Within n-ary relationship* $R(X_1, X_2, \ldots, X_n)$ *with functional dependencies between n attributes determined by formula* (15)*, there may exist a functional dependency* $U - \{X_i, X_j\} \rightarrow X_i$*, where* $i \neq j$*,* $i, j = 1, 2, \ldots, n$*, if attribute* X_i *occurs in the right side of any FD* (15).

Let us consider a multiargument relationship of the scheme $R(X_1, X_2, \ldots, X_n)$ with the following FFDs:

$$U - \{X_i\} \rightarrow_{\alpha_{i,X},\, \beta_{X_i}} X_i, \quad i = 1, 2, \ldots, m\,, \quad m \leq n\,, \tag{17}$$

where

$$\alpha_{i,X} = (\alpha_{i,X_1}, \ldots, \alpha_{i,X_{i-1}}, \alpha_{i,X_{i+1}}, \ldots, \alpha_{i,X_n}) \tag{18}$$

Let us define the following fuzzy sets of attributes:

$$\mathcal{L} = \{\theta_1/X_1,\ \theta_2/X_2,\ \ldots, \theta_n/X_n\}\,, \quad \mathcal{B} = \{\beta_{X_1}/X_1,\ \beta_{X_2}/X_2,\ \ldots., \beta_{X_n}/X_n\}\,, \tag{19}$$

where

$$\theta_i = \begin{cases} (\sum_{j=1}^{j=m} \alpha_{j,\, X_i})/\, m & \text{if } \beta_{X_i} = 0 \\ (\sum_{j=1}^{j=m} \alpha_{j,\, X_i})/\, (m-1) & \text{if } \beta_{X_i} > 0 \text{ and } m > 1 \\ 0 & \text{if } \beta_{X_i} > 0 \text{ and } m = 1\ , \end{cases} \tag{20}$$

Theorem 2. *Within the fuzzy n-ary relationship* $R(X_1, X_2, \ldots, X_n)$ *with FFDs* (17) *there may exist* (*n-1*)*-ary relationships determined by the following FFDs:*

$$U - \{X_i, X_j\} \rightarrow_{\phi_{i,j,X},\ \lambda_{X_i}} X_i, \quad i \neq j, \quad i, j = 1, 2, ..., n, \tag{21}$$

where

$$\phi_{i,j,X} = (\phi_{i,j,X_1}, ..., \phi_{i,j,X_{i-1}}, \phi_{i,j,X_{i+1}}, ..., \phi_{i,j,X_{j-1}}, \phi_{i,j,X_{j+1}}, ..., \phi_{i,j,X_n})\ , \tag{22}$$

in which $\forall_{k\neq i,\ k\neq j}\ \phi_{i,j,X_k} \geq \alpha_{i,X_k}$ *and* $\lambda_{X_i} \leq \beta_{X_i}$.

Proof. The possibility to impose (n-1)-ary relationships is limited by values of $\alpha_{i,X}$ and β_{X_i}. The membership degrees of attributes in the sets (19) cannot be changed. Due to the Armstrong's rules: $U-\{X_i, X_j\}\rightarrow_{\phi_{i,j,X},\ \lambda_{X_i}} X_i \Rightarrow U-\{X_i\}\rightarrow_{\phi_{i,j,X},\ \lambda_{X_i}} X_i$ and $U-\{X_i\}\rightarrow_{\alpha_{i,X},\ \beta_{X_i}} X_i \Rightarrow U-\{X_i\}\rightarrow_{\phi_{i,j,X},\ \lambda_{X_i}} X_i$ for $\phi_{i,j,X_k} \geq \alpha_{i,X_k}$, $k \neq i$ and $k \neq j$, and $\lambda_{X_i} \leq \beta_{X_i}$. Thus, levels of FFDs (16) remain the same as well as the fuzzy sets (19). The imposed relationship does not disturb the integrity conditions. □

5. Conclusions

The subject of considerations presented in the paper is the possible occurrence - within the n-ary relationship with attributes represented by means of possibility distributions - of fuzzy functional dependencies (21) describing the relationships between (n-1) attributes. The starting point are fuzzy functional dependencies (17) existing between all attributes of the relation scheme $R(X_1, X_2, \ldots, X_n)$. In the paper we applied the definition of FFD which was formulated by Cubero [16]. Its level is evaluated by closeness measures (6) for all attributes. Dependencies (21) are admissible if their levels satisfy conditions which have been formulated in Theorem 2.

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