JOURNAL OF APPLIED
COMPUTER SCIENCE
Vol. 24 No. 3 (2016), pp. 49-60

Interval Model of Dynamics of Dispersion of Harmful Pollution from Vehicular Traffic

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Abstract. *The method of structure identification of interval discrete dynamic models, based on principles of the bee colonies functioning is represented. An example of the implementation of method for modeling of air pollution by harmful vehicle emissions is considered.*
Keywords: *structure identification, interval discrete dynamic models, artificial bee colony algorithm, dispersion of harmful pollution, vehicular traffic.*

1. Introduction

Recently in large cities especially relevant is the problem of environmental pollution by exhaust of vehicles, including nitrogen dioxide, carbon monoxide, sulfur oxides, dust and others. One of the ways of solving the mentioned problem is the widespread use of hybrid vehicles. However, cars that use internal combustion engines for a long time will pollute the environment of cities [1, 2] and the problem of minimizing harmful emissions of vehicles will be remain relevant.

As you know, the theoretical basis for modeling the propagation of harmful substances pollution in the atmosphere is serving as differential equations or difference analogues [2]. Therefore, when it comes to the dynamics of concentrations of harmful substances at some point, the modeling of this class of processes will perform based on discrete dynamical models, the structure of which will be selected on the conditions of agreement with experimental data. Besides that, because of high observational errors, the bounds of which are usually known, discrete dynamic models will be build based on methods of interval data analysis. The most significant results in the research of structure identification of mathematical models are obtained in the works [3, 4, 5, 6]. The basis of the often used methods of structure identification of macro models in kind of discrete dynamic models are criteria which are based on minimizing the standard deviation between predicted and experimental data. In conditions of large errors of observations, this approach is unacceptable. In addition, such known methods as “augmentation” or reduction of macro model structure and combinatorial methods of finding the optimal structure lead to significant increasing of structure complexity or very high computational complexity [7, 8].

For solving the tasks of structure identification the algorithms of foraging behavior of bee colony are widely used [9]. Such models are used in the construction of algorithms of optimization tasks solving when the objective function is much extreme or highly noised or discrete. Tasks of structure identification of models by the nature belong to this class. It is therefore advisable to apply artificial bee colony algorithm (ABCA) for realization the method of structure identification of discrete models of dynamics of concentrations of harmful emissions on the basis of interval data. Such models will call interval discrete dynamic models (IDDM).

2. Statement of the problem

We will describe the macro models that represent temporal distribution of concentrations of harmful substances by linear difference operator in such general form:

$$v_{k+1} = \vec{g}^{T} \cdot \vec{f}(v_0, ..., v_k, u_0, ..., u_k), k = 0, ..., N - 1, \tag{1}$$

where $\vec{f}(v_0, ..., v_k, u_0, ..., u_k)$ is unknown vector of basis functions that defines the structure of difference operator; v_{k+1} is predicted value of concentrations of harmful emissions (by type of chemical substance) at the moment of time $k+1$; $\vec{u}_k = (u_0, ..., u_k)^T$ is known vector of input variables (vehicle traffic intensity) at the discrete moment of time k; $\vec{g}$ is unknown vector of parameters.

For estimation vector of parameters $\vec{g}$ the results of observations at the moment of time k are using, which will present as a model with additive error:

$$\tilde{v}_k = c_k \cdot v_k + e_k \quad (2)$$

where $\tilde{v}_k$ is value of observed characteristics in moment of time k (measured value of carbon monoxide concentration); c_k is known coefficient that determines the characteristics of the measuring device; e_k are random, limited in amplitude errors

$$e_k \leq \Delta_k, \Delta_k > 0 \forall k = 0, ..., N-1. \quad (3)$$

As can be seen, in general the errors depend from the time of measurement. Using measurement model (2) and taking into account the limited in amplitude error (3), experimental data are represented in interval form:

$$v_k = [v_k^-; v_k^+] = [(\tilde{v}_k - \Delta_k); (\tilde{v}_k + \Delta_k)]/c_k, k = 0, ..., N-1. \quad (4)$$

where $[v_k^-; v_k^+]$ is guaranteed interval of measured value of concentrations of harmful emissions.

Unknown vector of parameters of difference operator $\vec{g}$ will be estimated in conditions of inclusion the predicted values corresponding interval of experimental data. Mentioned conditions are formally represented in such form:

$$\hat{v}_{k+1} = [\hat{v}_{k+1}^-; \hat{v}_{k+1}^+] \subseteq v_{k+1} = [v_{k+1}^-; v_{k+1}^+], k = 0, ..., N-1. \quad (5)$$

where $\hat{v}_{k+1} = [\hat{v}_{k+1}^-; \hat{v}_{k+1}^+]$ is predicted interval that in general is calculated using the equation:

$$[\hat{v}_{k+1}] = \hat{\vec{g}}^T \cdot \vec{f}([\hat{v}_0], ..., [\hat{v}_k], u_0, ..., u_k), k = 0, ..., N-1, \quad (6)$$

where $\hat{\vec{g}}$ is the vector of estimates of IDDM parameters which will be obtained using inclusion conditions (5), and $[\hat{v}_0], ..., [\hat{v}_k]$ are given or calculated interval estimates of initial discrete values of predicted concentrations of harmful substances.

Because for obtaining the interval of predicted concentrations of harmful substances $\hat{v}_{k+1}$ using equation (6) it is necessary to perform the calculations using interval arithmetic rules, then this equation is IDDM of concentrations of harmful substances.

Substituting interval estimates $[\hat{v}_{k+1}]$ calculated by equation (6) in presence of initial approximations $[\hat{v}_0], ..., [\hat{v}_k]$ into equation (5), we obtain the following interval system of nonlinear algebraic equations (ISNAE) [6]:

$$v_{k+1}^{-} \leq \hat{\vec{g}}^T \cdot \vec{f}([\hat{v}_0], ..., [\hat{v}_k], u_0, ..., u_k) \leq v_{k+1}^{+}, k = 0, ..., N-1, \quad (7)$$

Note that obtained interval system of algebraic equations will be linear for static systems [10]. In this case the task of structure identification can be solved using the algorithm given in the paper and estimations of model parameters [10] are multiple.

However, in the case of IDDM structure identification we have ISNAE (7) and mostly are looking for its one solution from the whole set.

We will consider that the IDDM structure is not known, and its structural elements are formed of the following subsets: $U_p^s = \{\vec{u}_k \in R^p | \{u_0, ..., u_k\}\}$ is set of input variables (controls); $V_\eta^s = \left\{\hat{\vec{v}}_k \in R^\eta | \{\hat{v}_0, ..., \hat{v}_k\}\right\}$ is set of known discrete values of predicted concentrations of harmful substances, which characterizes the order of IDDM; $G_m^s = \{\vec{g}_m \in R^m | \{g_1, ..., g_m\}\}$ is set of IDDM parameters; $F_{m,d}^s = \left\{\vec{f} \in R^m | \left\{f_1([\hat{\vec{v}}_k], \vec{u}_k), ..., f_m([\hat{\vec{v}}_k], \vec{u}_k)\right\}\right\}$ is set of basis functions of IDDM; p is the number of input variables; η is order of IDDM; m is the number of basis functions and the number of IDDM parameters. So, the current IDDM structure can be described in form of such tuple:

$$\lambda_s : \left\langle U_p^s, V_\eta^s, G_m^s, F_{m,d}^s \right\rangle \quad (8)$$

Structural elements from set U_p^s, V_η^s of mentioned tuple are associated by vector of basis functions. The task of structure identification is to find a tuple λ_s in the form (8), which ensures compatibility of ISNAE (7). Obviously, the compatibility of this system can be achieved by more complex IDDM structures, which is unacceptable in terms of macro modeling. At the same time, in paper [11] has been considered the indicator of the structure quality assessment in the following form:

$$\begin{aligned} \delta(\lambda_s) = \quad & \max_{k=0,...,N-1} \{|mid([\hat{v}_{k+1}(\lambda_s)]) - mid([v_{k+1}])|\}, \\ & if [\hat{v}_{k+1}(\lambda_s)] \cap [v_{k+1}] = \varnothing, \exists k = 0, ..., N-1, \end{aligned} \quad (9)$$

$$\begin{aligned} \delta(\lambda_s) = \quad & \max_{k=0,...,N-1} \{wid([\hat{v}_{k+1}(\lambda_s)]) - wid([\hat{v}_{k+1}(\lambda_s)] \cap [v_{k+1}])\}, \\ & if [\hat{v}_{k+1}(\lambda_s)] \cap [v_{k+1}] \neq \varnothing, \exists k = 0, ..., N-1, \end{aligned} \quad (10)$$

where $mid(\cdot), wid(\cdot)$ are the operations of determining of center and the width of the interval, respectively.

Then the general statement of the problem of structure identification is formulated in kind of the following optimization problem:

$$\delta(\lambda_s) \xrightarrow{\lambda_s} min \tag{11}$$

The lower value of $\delta(\lambda_s)$, the "better" current IDDM structure. If $\delta(\lambda_s) = 0$, the current IDDM structure provides an opportunity to build an adequate model.

3. Method of IDDM structure identification

Synthesis of IDDM structure (6) for the prediction of concentrations of vehicle harmful emissions in the atmosphere will conduct using a known method of structure identification, which built based on ABCA [11]. ABCA models the foraging behavior of bee colony [9, 12]. The essence of the algorithm is as the following: the better food source quality, the more bees flying there; bees leave exhausted food sources and fly to look for "new" ones in a random direction [13, 14]. All bees at the colony are conventionally divided into three groups: the worker-bees, explorer-bees and scout-bees.

The application of the principles of behavioral models of honeybees enables to apply it in the method of structure identification of IDDM that ensure minimization goal function (14) in a way of transforming of a sequence of structures $\lambda_1, ..., \lambda_s, \lambda_{s+1}, ..., \lambda_0$ by using such operators: $P(\Lambda_{mcn}, F)$, $P_N(F, I_{min}, I_{max})$, $P_\delta(\Lambda_{mcn}, F)$, $D_1(\lambda_s, \lambda_s^{'})$, $D_2(\lambda_s, \Lambda_s^{'})$, which are the analogs to the corresponding procedures of behavioral model of bee colony during food sources searching [9].

Next, consider basic steps of iterative scheme of implementing this method of structure identification in more detail.

Step 1. Initialization of initial parameters of the algorithm. Set the values of the following variables: MCN, $LIMIT$, S, $[I_{min}; I_{max}]$ and a set of structural elements F. Then forming the initial set of IDDM structures Λ_0 (with power S). It should be noted that the initial set of IDDM structures Λ_0 is forming randomly from a set of structural elements F.

Step 2. Synthesis of set of current IDDM structures $\Lambda_{mcn}^{'}$ is carried out by transformation of set of structures Λ_{mcn} (mcn is number of the current iteration) into a set of structures $\Lambda_{mcn}^{'}$. This transformation is executed by using the operator

$P(\Lambda_{mcn}, F)$. Then conduct a pairwise selection of IDDM structures using operator $D_1(\lambda_s, \lambda_s^{'})$ and obtain the set of "best" structures Λ_{mcn}^1.

Note that the operator $P(\Lambda_{mcn}, F)$ implements the investigation procedure of neighborhood of known nectar source by worker-bees of the colony, and pairwise selection operator $D_1(\lambda_s, \lambda_s^{'})$ implements the procedure of memorizing of coordinates and a nectar source quality for worker-bees from behavioral model of bee colony.

Step 3. Transformation of each structure λ_s^1 from a set of structures Λ_{mcn}^1 into a set of structures $\Lambda_s^{'}$ (where $s = 1, \ldots, S$) in the way of random replacing the elements of each structure Λ_s^1 by elements from the set F. Then denote by $\Lambda_{mcn}^{''}$ such union of sets $\Lambda_{mcn}^{''} = \left\{\Lambda_1^{'} \cup \Lambda_2^{'} ... \cup ...\Lambda_s^{'} ... \cup \Lambda_S^{'}\right\}, s = 1, ..., S$.

Thus, using operator $P_\delta(\Lambda_{mcn}, F)$ will obtain the set of structures $\Lambda_{mcn}^{''}$. Then conduct group selection of current IDDM structures using the operator $D_2(\lambda_s, \Lambda_s^{'})$ and obtain a set of "best" structures Λ_{mcn}^2 from the current sets Λ_{mcn}^1 and $\Lambda_{mcn}^{''}$.

Note that the operator $P(\Lambda_{mcn}, F)$ implements the investigation procedure of neighborhood of known nectar source by explorer-bees, and group selection operator $D_2(\lambda_s, \Lambda_s^{'})$ implements the procedure of memorizing of coordinates and a nectar source quality for explorer-bees from behavioral model of bee colony.

Step 4. Checking of "exhaustion" of current IDO structures.

All the structures $\lambda_s^2 \in \Lambda_{mcn}^2$ for which the condition $Limit_s \geq LIMIT$ is met, are "exhausted". If the structure is "exhausted", then it is replaced by "new" one which is generated based on the operator $P_N(F, I_{min}, I_{max})$.

The operator $P_N(F, I_{min}, I_{max})$ generates in a random way "new" IDDM structure from a set of all structural elements according to the procedure of random search of new nectar source at the behavioral model of bee colony. If found at least one structure, for which $\delta(\lambda_s^2) = 0$, the procedure of structure identification is completed, otherwise go back to step 2. Mentioned transformations implement phases of activity of all bee groups from ABCA.

4. Application of the method of structural identification for construction IDDM of carbon monoxide concentrations depending on the intensity of transport streams

Data for the construction of IDDM were received during performance of national research project on the topic "Mathematical tools and software for control the air pollution from vehicles" (0116U005507).

The dynamics of carbon monoxide concentrations was set by measurements of its concentrations on streets crossing "Brodivska - Zbarazka - Dovha" in Ternopil (Ukaine) on April, 7, 2015. The mentioned crossroads are marked by enough high traffic of vehicles, as well as by the average wind.

Accuracy of measuring of carbon monooxide concentrations by a spectroanalyzer as "SB-26" was 25 percent.

Table 1 displays the received interval estimates of carbon monoxide concentrations and also the intensity of vehicular traffic.

To realize the method of structure identification of macromodels in kind of IDDM and for modeling daily cycle of changes of carbon monoxide concentrations, will use the algorithm described above.

It should also be noted that considered method of structure identification based on solving an optimization task with objective function (9) or (10), which it is known is not unimodal and also can reach the equivalent global minimums at different points (for different structures of models). Therefore, the "best" IDDM received by applying the algorithm, that was discussed above, may have a different structure and even the same complexity in the sense of the number of elements.

The set of structural elements using a polynomial functions not higher the second degree and for IDDM not higher the second order was generated. As a result, it was received a set of structural elements by power of $L = 45$ that is given in Table 2.

Then establish the value of initial parameters of the algorithm: $I_{min} = 3$, $I_{max} = 5$, MCN=100, $LIMIT$=4, S=10. After that, randomly form the initial set of IDDM structures Λ_0 ($S = 10$) that are given in Table 3.

During researches on the third iteration of application of method of structure identification based on ABC it was received adequate structures of macromodel: $\lambda_0 = \{16, 18, 22, 45\}$, $\lambda_0 = \{3, 9, 21\}$, for which a condition is executed $\delta(\lambda_0 = 0)$. Formed on the basis of found structures λ_0 of IDDM has such kind:

$$\hat{v}_{k+1} = 0.32418 - 0.63804 \cdot \hat{v}_k \cdot u_k/u_{k-1} - 0.29226 \cdot \hat{v}_k \cdot u_k/u_{k+1} + \\ +0.21413 \cdot \hat{v}_{k-2} \cdot u_{k-2}/u_k + 0.33522 \cdot \hat{v}_{k-1} \cdot u_{k+1}/u_k \quad (12)$$

$$\hat{v}_{k+1} = -0.52637 + 0.19304 \cdot \hat{v}_k + 0.02816 \cdot \hat{v}_k \cdot \hat{v}_{k-1} + 0.75903 \cdot \hat{v}_k \cdot u_{k+1}/u_k \quad (13)$$

As we can see, the models (12) and (13) keeps the logic of daily cycle of change of harmful emissions depending on intensity of vehicular traffic.

Results of modeling of daily cycle of change of the carbon monoxide concentrations of harmful emissions on crossing of streets "Brodivska - Zbarazka -

Table 1: Output data for dynamics modeling of carbon monoxide concentration

Time, $hours, t$	$v_k^-, mg/m^3$	$v_k^+, mg/m^3$	$u_k, cars$
01:00:00	2,2	3,7	853
02:00:00	1,9	3,2	721
03:00:00	2,1	3,5	634
04:00:00	2,2	3,7	763
05:00:00	2,6	4,3	972
06:00:00	4,2	7	1831
07:00:00	5	8,3	2675
08:00:00	5,2	8,7	2707
09:00:00	4,7	7,8	2517
10:00:00	5,8	9,7	2834
11:00:00	4,9	8,2	2528
12:00:00	4,7	7,8	2476
13:00:00	5,6	9,3	2875
14:00:00	4,2	7	2178
15:00:00	5,3	8,8	2714
16:00:00	5,6	9,3	2635
17:00:00	4,7	7,8	2406
18:00:00	4,5	7,5	2382
19:00:00	4,1	6,8	2152
20:00:00	3,5	5,8	1934
21:00:00	2,8	4,7	1303
22:00:00	2,7	4,5	1056
23:00:00	2,6	4,3	878
24:00:00	2,3	3,8	861

Dovha" in the city Ternopil (Ukraine) with using IDDM in kind (12) and (13) are given on Figures 1 and 2 respectively.

Apparently, the predicted dynamics of daily cycle of changes of carbon monoxide concentrations for both models is shown by the dotted line, is within experimental data and determines the given carbon monoxide concentration within the errors of observation.

Table 2: Set of structural elements of F

N	Structural element	N	Structural element	N	Structural element
1	v_{k-2}	16	$v_k \cdot u_k/u_{k-1}$	31	$v_{k-2} \cdot u_{k+1}/u_{k-1}$
2	v_{k-1}	17	$v_k \cdot u_k/u_{k-2}$	32	$v_{k-2} \cdot u_{k+1}/u_{k-2}$
3	v_k	18	$v_k \cdot u_k/u_{k+1}$	33	$v_{k-2} \cdot u_{k+1}/u_k$
4	v_{k-2}^2	19	$v_k \cdot u_{k+1}/u_{k-1}$	34	$v_{k-1} \cdot u_{k-2}/u_k$
5	v_{k-1}^2	20	$v_k \cdot u_{k+1}/u_{k-2}$	35	$v_{k-1} \cdot u_{k-2}/u_{k-1}$
6	v_k^2	21	$v_k \cdot u_{k+1}/u_k$	36	$v_{k-1} \cdot u_{k-2}/u_{k+1}$
7	$v_{k-2} \cdot v_k$	22	$v_{k-2} \cdot u_{k-2}/u_k$	37	$v_{k-1} \cdot u_{k-1}/u_k$
8	$v_{k-2} \cdot v_{k-1}$	23	$v_{k-2} \cdot u_{k-2}/u_{k-1}$	38	$v_{k-1} \cdot u_{k-1}/u_{k-2}$
9	$v_{k-1} \cdot v_k$	24	$v_{k-2} \cdot u_{k-2}/u_{k+1}$	39	$v_{k-1} \cdot u_{k-1}/u_k$
10	$v_k \cdot u_{k-2}/u_k$	25	$v_{k-2} \cdot u_{k-1}/u_k$	40	$v_{k-1} \cdot u_k/u_{k-1}$
11	$v_k \cdot u_{k-2}/u_{k-1}$	26	$v_{k-2} \cdot u_{k-1}/u_{k-2}$	41	$v_{k-1} \cdot u_k/u_{k-2}$
12	$v_k \cdot u_{k-2}/u_{k+1}$	27	$v_{k-2} \cdot u_{k-1}/u_{k-2}$	42	$v_{k-1} \cdot u_k/u_{k+1}$
13	$v_k \cdot u_{k-1}/u_k$	28	$v_{k-2} \cdot u_k/u_{k-1}$	43	$v_{k-1} \cdot u_{k+1}/u_{k-1}$
14	$v_k \cdot u_{k-1}/u_{k-2}$	29	$v_{k-2} \cdot u_k/u_{k-2}$	44	$v_{k-1} \cdot u_{k+1}/u_{k-2}$
15	$v_k \cdot u_{k-1}/u_k$	30	$v_{k-2} \cdot u_k/u_{k+1}$	45	$v_{k-1} \cdot u_{k+1}/u_k$

Table 3: Initial set of IDDM structures

N	Ordinal numbers of structural elements from the set of F, that define structures λ_s, $s = 1, ..., S$.	$\delta(\lambda_s)$
1	14, 22, 27, 40	1,6
2	4, 12, 30, 32, 43	0,726
3	2, 13, 22	1,348
4	6, 9, 28, 32	0,883
5	8, 11, 17, 20	2,546
6	7, 19, 41	1,475
7	15, 26, 38, 44	3,34
8	2, 8, 25	3,081
9	12, 23, 25, 30, 33	4,892
10	1, 16, 19	2,073

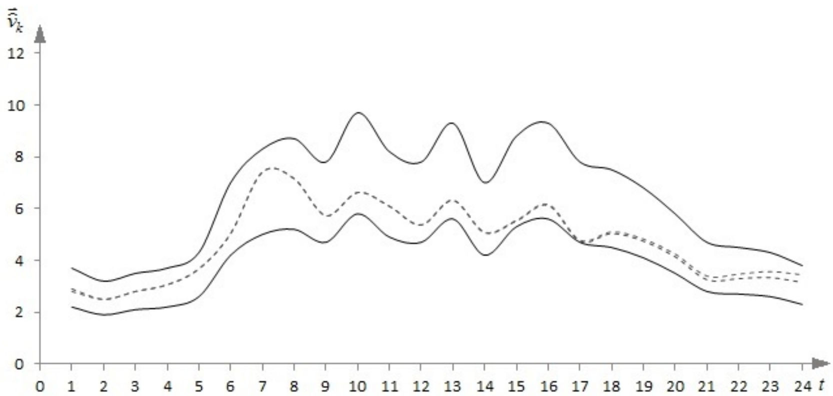

Figure 1: The dynamics of daily cycle of change of carbon monoxide concentrations based on IDDM (12)

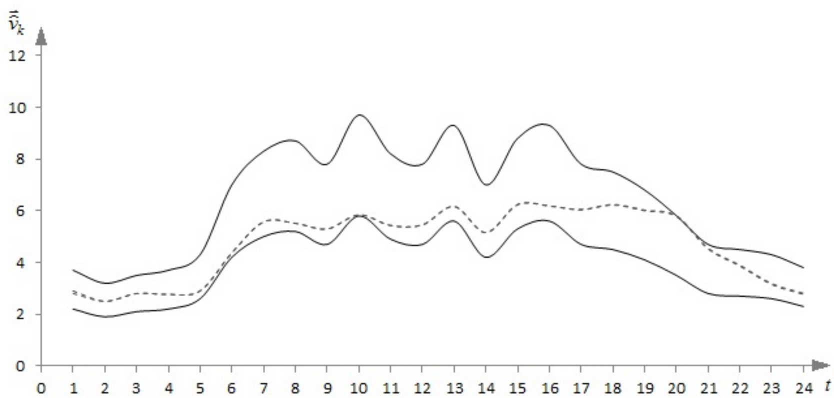

Figure 2: The dynamics of daily cycle of change of carbon monoxide concentrations based on IDDM (13)

5. Conclusions

The task of structure identification of IDDM of harmful emissions concentrations from vehicles is researched. For solving this task it is proposed to use the method of structure identification of IDDM built on the basis of algorithm of bee colony. It was conducted the research of dynamics change of harmful emissions concentrations in kind carbon monoxide during day and night on streets crossing of the city of Ternopil. Comparison of the predicted concentrations with experimental concentrations within measurement errors confirms the adequacy of the developed mathematical model.

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