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SPIN WAVES PARAMETER IN MAGNETIC NANODOTS

The spin waves parameter B describing low temperature behaviour of magnetisation has been calculated for the array of magnetic dots. The Heisenberg model has been used with the exchange and shape anisotropies taken into account. The dependence of B on the dot size and interaction parameters has been found. The results obtained are in agreement with experimental findings.

Keywords: magnetic dots, spin waves parameter, Bloch's law.

1. INTRODUCTION

Nano sized ferromagnetic elements are attracting considerable attention due to numerous potential applications such as high density memory devices and sensors [1-3]. Modern techniques for film deposition and lithography make it possible to fabricate arrays of small magnetic particles. Investigation of magnetic properties of the arrays allow one to study fundamental magnetic properties of low dimensional ferromagnets [4-7] as it is known that the reduction in dimension changes the magnetic properties compared to infinite films [8]. In this paper we focus our attention on the low temperature behavior of magnetization and thermal spin excitations in cuboidal dots arrays

2. MODEL AND STRUCTURE

Presently the necessity of dealing with particles of ellipsoidal shapes in order to have the uniform demagnetization field inside them can be avoided for experimentally accessible range. For example, the problem of magnetic

properties of nanowires was investigated theoretically [9] by treating nanowire as an object with square or elliptical cross section and third dimension much greater than cross section diameter. We apply this simple model to the case of array of magnetic cubes of nanoscopic dimensions with proper boundary conditions appearing for corners end edges of nanostructures. In dependence of sizes of such structures one needs to take into consideration the dominant role either of the dipolar interactions between spins in the dot (for larger particles) or of an exchange type interaction (for smaller particles). The distance between dots is chosen in such a way that the dipolar coupling between them can be neglected [10].

The exchange part of Hamiltonian describing the system under consideration contains all interactions between neighboring pairs of Heisenberg type spins positioned in N x N x N lattice points, where N denotes the dimension of cube in lattice constants [9, 11].

$$\mathcal{H}_{ex} = -J_1 \sum_{j,n=1}^{N} \sum_{k=1}^{N-1} \left(S_{j,k,n}S_{j,k+1,n} + S_{j,k+1,n}S_{j,k,n}\right) + -J_2 \sum_{k,n=1}^{N} \sum_{j=1}^{N-1} \left(S_{j,k,n}S_{j+1,k,n} + S_{j+1,k,n}S_{j,k,n}\right) + -J_3 \sum_{j,k=1}^{N} \sum_{n=1}^{N-1} \left(S_{j,k,n}S_{j,k,n+1} + S_{j,k,n+1}S_{j,k,n}\right). \quad (1)$$

In formula (1) J_1, J_2, and J_3 are the exchange parameters in x, y, z direction, respectively. $J_1=J_2=J_3$ means completely isotropic system from the point of view of exchange interaction. The more realistic assumption that $J_2 = J_1$ and $J_3 = J1(1 + \Delta)$, where $\Delta \in <-1,1>$ introduces to the system the exchange type anisotropy in the z direction. The term describing surface type anisotropy in interaction Hamiltonian is represented by the single ion anisotropy and is connected to the existence of edges in nanodots. Taking D_1, D_2, D_3 as anisotropy parameters in the x, y, z direction one can write the anisotropy term in the following form [9, 11]:

$$\mathcal{H}_{anis} = -D_1 \sum_{k,n=1}^{N} [(S_{1kn}^{x})^2 + (S_{Nkn}^{x})^2] - D_2 \sum_{j,n-1}^{N} \left[\left(S_{j1n}^{y}\right)^2 + \left(S_{jNn}^{y}\right)^2\right] + - D_3 \sum_{j,k=1}^{N} \left[\left(S_{jk1}^{z}\right)^2 + \left(S_{jkN}^{z}\right)^2\right]. \quad (2)$$

The size and shape of nanoparticles require introducing also the shape anisotropy term to the total Hamiltonian. It can be done in the way proposed in [9]:

$$\mathcal{H}_{dip} = D_{dip} \sum_{j,k,n=1}^{N} \left(S_{jkn}^{z}\right)^2 , \tag{3}$$

where $D_{dip} = n_d M_s$ is the shape anisotropy coefficient along the z axis, with n_d denoting the demagnetizing factor and M_s the saturation magnetization in the same direction. Interaction Hamiltonian contains also the Zeeman part, namely

$$H_{Zeeman} = -g\mu_B H \sum_{jkn=1}^{3} S_{jkn}^{z}. \tag{4}$$

3. METHOD OF CALCULATION

For the system considered the demagnetizing field can be calculated in the other way to omit the problem of unknown parameter D_{dip}. In this approach [12] the structure is treated as an object consisting of parallel planes with magnetic moments μ_r arranged regularly in a crystal lattice. The dipolar energy in reference point **r** comes from contribution from each lattice plane and the magnetic field $\mathbf{h_R}$ derived from the prism dipoles indicated by vector **R** can be described in the classical form:

$$\boldsymbol{h_r} = \frac{1}{4\pi} \sum\nolimits_{r \neq R} \frac{3(\boldsymbol{r}-\boldsymbol{R})\left(\mu_r \cdot (\boldsymbol{r}-\boldsymbol{R})\right) - \mu_r |\boldsymbol{r}-\boldsymbol{R}|^2}{|\boldsymbol{r}-\boldsymbol{R}|^5} \tag{5}$$

Denoting lattice points by *p, q* $\in$*<-L,L>* and planes by *n* $\in$ *<0,L-1>,* where for symmetry reason *N* is chosen as equal to *2L + 1*, one can after some transformations formula for magnetic caused by dipolar interaction in the plane *n'*:

$$\boldsymbol{h}_{n'} = \frac{1}{4\pi} \sum_{n} D_{n,n'} \left[\hat{\imath} M_n^x + \hat{\jmath} M_n^y - 2\hat{k} M_n^z ,\right] \tag{6}$$

with $D_{n,n'}$ defined as

$$D_{n,n'} = \sum_{p,q} \frac{\frac{1}{2}(p^2 + q^2) - (n-n')^2}{(p^2 + q^2) + (n-n')^2} . \tag{7}$$

Finally the shape anisotropy Hamiltonian (3) can be replaced by

$$\mathcal{H}_{dip} = -D_{dip} \sum_{j,k,n=1}^{N} \left(S_{jkn}^{z}\right)^2 \implies -g\mu_B \sum_{jkn'=1}^{N} h_{n'}^{z} S_{jkn}^{z} . \tag{8}$$

In order to calculate low temperature properties of the system it is convenient to express the spin operators to the second quantisation Bose operators b^+ and b employing the Holstein-Primakoff transformation [13]:

$$S^+ = \sqrt{2S}\, b\,, \qquad S^- = \sqrt{2S}\, b^+\,, \qquad S^z = S - b^+ b, \tag{9}$$

$$S^x = \frac{1}{2}\,(S^+ + S^-)\,, \qquad S^y = -\frac{i}{2}\,(S^+ - S^-)\,, \tag{10}$$

$$\tfrac{1}{2}(S^+)^2 = S\,(b)^2\,, \qquad \tfrac{1}{2}(S^-)^2 = S\,(b^+)^2 \tag{11}$$

$$\tfrac{1}{2}S^+S^- = Sbb^+\,, \qquad \tfrac{1}{2}S^-S^+ = Sb^+b \tag{12}$$

$$(S^z)^2 = -(2S-1)\,b^+b + \text{terms with higher powers in } b \text{ and } b^+. \tag{13}$$

Applying formulae (9) (13) to of total Hamiltonian consisting of parts described by formulas (1), (2), (4) and (8) one can obtain the matrix of coefficients at the b^+b terms. For example for the cube of L x L x L spins with exchange parameters in all direction equal, the matrix $\|P_{jkn,stv}\|$ can be expressed in terms of Rutherford matrices [14] in the form [9]:

$$\|P_{jkn,stv}\| = R_L^2\left\{R_L^2(R_L^1[-6+h, a_1, a_1], a_2 I_L, a_2 I_L), a_2 I_L, \otimes I_L, a_2 I_L \otimes I_L\right\}, \tag{14}$$

where I_L denotes the L x L unit matrix and $R_L(x,a,b)$ matrix is defined by:

$$R_\xi(x,a,b) = \begin{bmatrix} x+a & 1 & & & & & \\ 1 & x & 1 & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ & & 1 & x & 1 & & \\ & & & 1 & x & 1 & \\ & & & & 1 & x+a & \end{bmatrix}_{(LxL)}. \tag{15}$$

In formula (14) the following notation has been used:

$$h = -\frac{\left[D_{dip}(2S-1) + g\mu_B H\right]}{2JS},$$

$$a_1 = 1 - d_1, \qquad a_2 = 1 + d_2, \qquad a_3 = 1 + d_3, \tag{16}$$

where

$$d_{1,2} = \frac{D_{1,2}S}{2JS}, \qquad d_3 = \frac{D(2S-1)}{2JS}\ . \tag{17}$$

Matrices of coefficients for the structure N_1 x N_2 x N_3 can be obtained in the same way. For clarity we omit here their detailed presentation.

Using the matrix $\|P_{jkn,stv}\|$ described above one can calculate the components of eigenvectors in direction *x, y, z* and find energies $E(k_i)$ of magnetic excitations of wave vectors k_i. The mean number of spin waves $< n_i >$ excited at temperature T is given by the Bose-Einstein distribution function:

$$\langle n_i \rangle \sim \frac{1}{exp[E(k_i)/k_B T] - 1}, \tag{18}$$

and for temperatures well below the Curie temperature the temperature dependence of spontaneous magnetisation magnetization $M(T)$ of the system consisting of N lattice point can be expressed in the form of Bloch's relation, namely

$$\sum_i \frac{\langle n_i \rangle}{N} \sim \frac{M(0) - M(T)}{M(0)} = BT^b. \tag{19}$$

where B is constant called spin wave parameter and b is the Bloch exponent, which is given by $b = 3/2$ for three-dimensional systems. The increase of spin wave parameter B with decreasing magnetic layer thickness has been discussed in a number of papers [15-17].

4. NUMERICAL RESULTS AND DISCUSSION

Experimental data obtained for dot arrays with uniaxial magnetic anisotropy [10] show that for such systems the Bloch $T^{3/2}$ law is valid in the wide range of dot diameters. Results obtained in our calculation are consistent with those results. However it is necessary to notice that the temperature region should be carefully chosen. The dependence of spin waves parameter B on the inverse particles size for cubic nanodot showed in Fig. 1 is in good agreement with results obtained in above mentioned experiment [10].

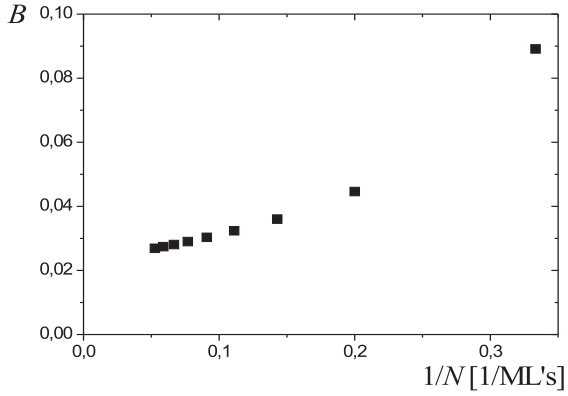

Fig. 1. The dependence of spin waves parameter B on the inverse size of cubic magnetic particle. Size of particle is given in numbers of magnetic monolayers.

For bigger structures dependence is of linear character, nonlinear behavior of B appears for smaller dots. This decay can be explained by larger influence of the spins on the surfaces and edges in comparison to the inner spins in smaller structures. The exchange energy per spin at the edges is much weaker than in the inside part, which leads to enhanced spin wave excitation and consequently to increase of spin wave parameter B. The magnetic characteristics can be calculated not only for nanoparticles of cubic shape. The nanodots with different ratio n_z/n, where n_z denotes the dimension in the z direction represent the special case of investigated structures. As we can expected, the results presented in Figs. 2 and 3 show similar behavior to results obtained for cubic systems.

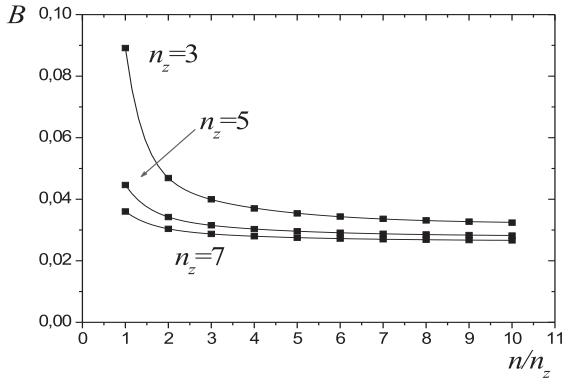

Fig. 2. Comparison of spin waves parameter B for different ratio of transversal to longitudinal dimension n/n_z.

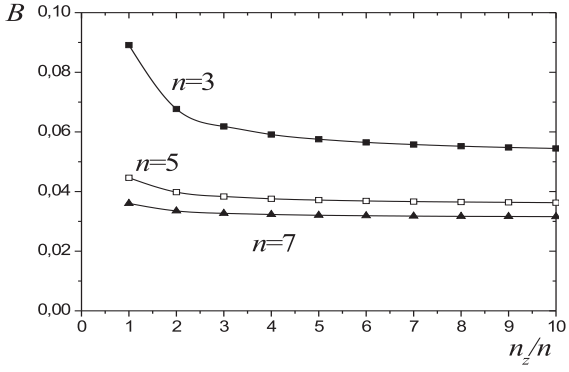

Fig. 3. Comparison of spin waves parameter B for different ratio of longitudinal to transversal dimension n_z/n.

In all low size structures the anisotropy parameter is very important characteristics of the system. For cubic small nanoparticles it is the anisotropy parameter D appearing because of existence of edges and corners. Positive and negative values of D correspond to the anisotropy in and out of plane, respectively. Results obtained for the case when $D_1 = D_2 = D_3 = D$ are presented in Fig. 4. For bigger nanodots the participation of corners and edges in energy of whole system is weaker than in smaller ones, so we can observe only small influence of this type energy on spin wave parameter B.

The exchange type anisotropy appears when the exchange parameters are different in different directions. In order to study the influence of that anisotropy on the behavior of low temperature magnetization the parameter Δ is introduced. as a normalized difference between the exchange interaction in the z direction and the interaction in the plane of structure ($\Delta \epsilon <-1,1>$). Fig. 5 presents the dependence of spin waves parameter B on Δ, for the case of cubic system with uniaxial exchange anisotropy,namely for $J_1 = J_2 = J$ and $J_3 = (1-\Delta)J$. One can see that the influence of such type of anisotropy on spin wave parameter B is important only for very small structures.

It is interesting to repeat described above procedure for parameters taken for material for whch exist experimental data showing dependence of parameters appearing in Bloch's law (19). There is experimental evidence that in the case of Fe nanodots the exponent $b = 3/2$ can be used in calculations [18]. Therefore we have used the following parameters describing iron nanodots [19]: $J = 0.4$ eV/atom and $D = 0.1$ eV/atom.The results plotted in Fig. 6 are in good agreement with experimental finding reported in [18].

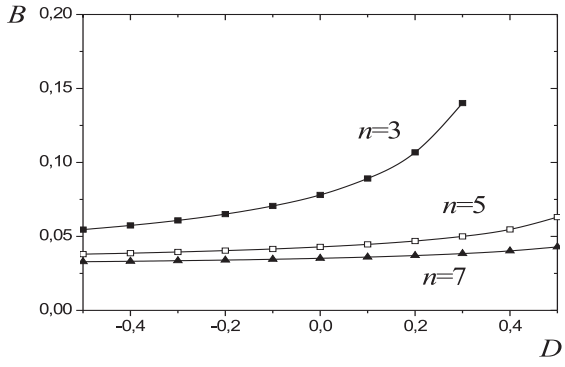

Fig. 4. Illustration ofthe influence of the anisotropy parameter D (normalized to 2JS) on spin wave parameter B.

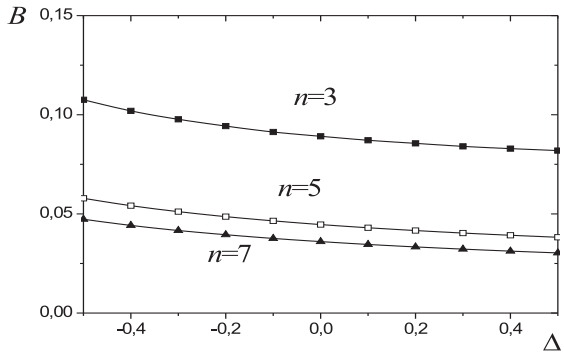

Fig. 5. Spin waves parameter B as a function of the exchange anisotropic interaction in chosen z direction represented by parameter Δ. $J_1 = J_2 = J$ and $J_3 = (1-\Delta)J$.

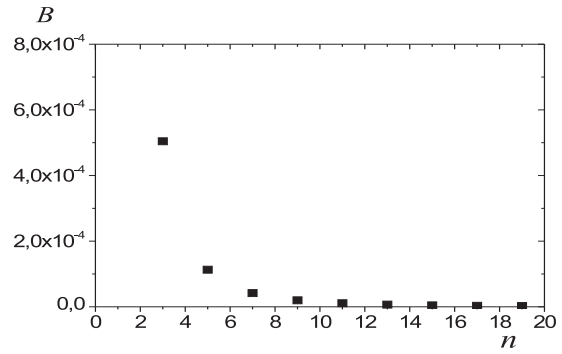

Fig. 6. The dependence of spin waves parameter B on the size of cubic iron particles. Size of particle is given in numbers of magnetic monolayers.

5. SUMMARY

Our results show that simple model of magnetic dots presented is this paper allows obtaining in spin wave region results which at least in qualitative way reproduce dependences observed in experiment studying properties of magnetic dots arrays. For the clarity of presentation we have focused on the behavior of spin waves parameter, formalism used in this paper allows also to obtain the spin wave profiles and consequently to calculate parameters of ferromagnetic resonance spectra for systems of magnetic dots. In calculations dipolar

interaction between dots has been neglected and results have been compared to the data obtained in experiments where that have been fulfilled. However the magnetic properties of dot arrays may be strongly affected by the interaction between the individual elements. Presented model should be therefore developed for the case when because of the size of edge to edge distance between dots the dipole interaction between them should be taken into account.

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PARAMETR FAL SPINOWYCH W UKŁADZIE NANOKROPEK MAGNETYCZNYCH

Streszczenie

Parametr fal spinowych *B* opisujący zachowanie się namagnesowania układu w niskich temperaturach został obliczony dla sieci kropek magnetycznych o nanoskopowych rozmiarach. Obliczenia były prowadzone przy użyciu modelu Heisenberga z uwzględnieniem anizotropii kształtu oraz anizotropii oddziaływań wymiennych. Znaleziono zależność *B* od rozmiarów kropek magnetycznych oraz od parametrów oddziaływania. Otrzymano wyniki zgodne z danymi doświadczalnymi.